# On fuzzy soft compact and fuzzy soft locally compact spaces

**Sabir Hussain**
*Department of Mathematics*
*College of Science*
*Qassim University*
*P. O. Box 6644*
*Buraydah 51482*
*Saudi Arabia*
*sabiriub@yahoo.com*
*sh.hussain@qu.edu.sa*

**Abstract.** In this paper, we define and explore the properties and characterization of fuzzy soft compact spaces at fuzzy soft point. We also initiate and investigate the concept of fuzzy soft locally compact spaces at fuzzy soft point through its properties and characterization. Examples of defined notions are also presented to validate their existence. We hope that the obtained results will proceed towards more natural findings and applications in real life problems with the issues of uncertainties and ambiguous environment.

**Keywords:** fuzzy soft sets, fuzzy soft topology, fuzzy soft open(closed), fuzzy soft classes, fuzzy soft compact, fuzzy soft locally compact.

## 1. Introduction

Fuzzy set theory initiated and studied by Zadeh [38] proved to be an important mathematical tool to solve different types of complicated problems having uncertainties in real life problems with ambiguous environment such as sociology, economics, engineering, computer and medical sciences etc. The real life problems associated with uncertainties are handled with probability theory, rough set theory, vague set theory, interval mathematics theory and fuzzy set theory. It is observed that these theories have their limitations and difficulties which require the pre-specification of some initial parameters to deal with them.

To overcome this situation of inadequacy of parametrization tools, Molodtsov [29] initiated soft sets as a modern and sufficient approach for modelling the complicated problems with uncertainties. In recent days, the application of soft set theory have been observed and studied by many researchers in the fields of information science, computer science, demand analysis, forecasting, engineering, medical science, social science and decision making [3], [7-8], [11], [13-14], [24], [26], [30-32].

Shabbir and Naz[34] defined and explored soft topological spaces. Later on, different algebraic structures, soft mappings and generalizations of soft sets have been discussed in [2], [5], [15-16], [21-23].

Maji and Biswas [27] introduced fuzzy soft sets as a generalization of soft sets. Z. Kong et. al [25] presented the application of fuzzy soft set as new theoretic approach to decision making problems. Chang [4] initiated and discussed the notion of fuzzy topological spaces. In [35-36], authors initiated fuzzy soft topology and discussed the algebraic structures of fuzzy soft topology. The applications and structural properties of fuzzy soft sets and fuzzy soft topology are discussed and explored in [1], [6], [9-10], [12], [17], [28], [33], [37].

Recently, S. Hussain [18-20] defined and investigated the properties and characterizations of generalized structures, generalized fuzzy soft mappings and weak and strong forms of fuzzy soft open sets in fuzzy soft topological spaces.

## 2. Preliminaries

**Definition 2.1** ([38])**.** A fuzzy set $f$ on $X$ is a mapping $f : X \to I = [0, 1]$. The value $f(x)$ represents the degree of membership of $x \in X$ in the fuzzy set $f$, for $x \in X$.

**Definition 2.2** ([29])**.** Let $X$ be an initial universe and $E$ be a set of parameters. Let $P(X)$ denotes the power set of $X$ and $A$ be a non-empty subset of $E$. A pair $(F, A)$ is called a soft set over $X$, where $F$ is a mapping given by $F : A \to P(X)$. In other words, a soft set over $X$ is a parameterized family of subsets of the universe $X$. For $e \in A$, $F(e)$ may be considered as the set of $e$-approximate elements of the soft set $(F, A)$.

**Definition 2.3** ([27])**.** Let $I^X$ denotes the set of all fuzzy sets on $X$ and $A \subseteq X$. A pair $(f, A)$ is called a fuzzy soft set over $X$, where $f : X \to I^X$ is a function. That is, for each $a \in A$, $f(a) = f_a : X \to I$, is a fuzzy set on $X$ .

**Definition 2.4** ([27])**.** For two fuzzy soft sets $(f, A)$ and $(g, B)$ over a common universe $X$, we say that $(f, A)$ is a fuzzy soft subset of $(g, B)$ if

(1) $A \subseteq B$ and

(2) for all $a \in A$, $f_a \leq g_a$; implies $f_a$ is a fuzzy subset of $g_a$.

We denote it by $(f, A)\tilde{\leq}(g, B)$. $(f, A)$ is said to be a fuzzy soft super set of $(g, B)$, if $(g, B)$ is a fuzzy soft subset of $(f, A)$. We denote it by $(f, A)\tilde{\geq}(g, B)$.

**Definition 2.5** ([27])**.** Two fuzzy soft sets $(f, A)$ and $(g, B)$ over a common universe $X$ are said to be fuzzy soft equal, if $(f, A)$ is a fuzzy soft subset of $(g, B)$ and $(g, B)$ is a fuzzy soft subset of $(f, A)$.

**Definition 2.6** ([27])**.** The union of two fuzzy soft sets of $(f, A)$ and $(g, B)$ over the common universe $X$ is the fuzzy soft set $(h, C)$, where $C = A \cup B$ and for all $c \in C$,

$$h_c = \begin{cases} f_c, & \text{if } c \in A - B, \\ g_c, & \text{if } c \in B - A, \\ f_c \vee g_c, & \text{if } c \in A \cap B. \end{cases}$$

We write $(f, A)\tilde{\vee}(g, B) = (h, C)$.

**Definition 2.7** ([27]). The intersection $(h, C)$ of two fuzzy soft sets $(f, A)$ and $(g, B)$ over a common universe $X$ , denoted $(f, A)\tilde{\wedge}(g, B)$, is defined as $C = A \cap B$, and $h_c = f_c \wedge g_c$, for all $c \in C$.

**Definition 2.8** ([27]). The difference $(h, C)$ of two fuzzy soft sets $(f, A)$ and $(g, B)$ over $X$, denoted by $(f, A)\tilde{\setminus}(g, B)$, is defined as $(f, A)\tilde{\setminus}(g, B) = (f, A)\tilde{\wedge}(f, B)^c$.

For our convenience, we will use the notation $f_A$ for fuzzy soft set instead of $(f, A)$.

**Definition 2.9** ([35]). Let $\tau$ be the collection of fuzzy soft sets over $X$, then $\tau$ is said to be a fuzzy soft topology on $X$ if

(1) $\tilde{0}_A$ , $\tilde{1_A}$ belong to $\tau$.

(2) If $(f_A)_i \in \tau$, for all $i \in I$, then $\tilde{\bigvee}_{i \in I}(f_A)_i \in \tau$.

(3) $f_a, g_b \in \tau$ implies that $f_a\tilde{\wedge}g_b\tilde{\in}\tau$.

The triplet $(X, \tau, A)$ is called a fuzzy soft topological space over $X$. Every member of $\tau$ is called fuzzy soft open set. A fuzzy soft set is called fuzzy soft closed if and only if its complement is fuzzy soft open.

**Definition 2.10** ([36]). Let $(X, \tau, A)$ be a fuzzy soft topological space over $X$ and $f_A$ be a fuzzy soft set over $X$. Then fuzzy soft closure of $f_A$ , denoted by $\overline{f_A}$ is the intersection of all fuzzy soft closed super sets of $f_A$. Clearly $\overline{f_A}$ is the smallest fuzzy soft closed set over X which contains $f_A$.

**Definition 2.11** ([1]). Let $F(X, A)$ and $F(Y, B)$ be families of fuzzy soft sets. $u : X \to Y$ and $p : A \to B$ be mappings. Then a function $f_{pu} : F(X, A) \to F(Y, B)$ defined as:

(1) Let $f_A$ be a fuzzy soft set in $F(X, A)$. The image of $f_A$ under $f_{pu}$ , written as $f_{pu}(f_A)$, is a fuzzy soft set in $F(Y, B)$ such that for $\beta \in p(A) \subseteq B$ and $y \in Y$,

$$f_{pu}(f_A)(\beta)(y) = \begin{cases} \bigvee_{x \in u^{-1}(y)}(\bigvee_{\alpha \in p^{-1}(\beta) \cap A}(f_A(\alpha)), & u^{-1}(y) \neq \phi, p^{-1}(\beta) \cap A \neq \phi \\ 0, & \text{otherwise} \end{cases},$$

for all $y \in B$. $f_{pu}(f_A)$ is known as a fuzzy soft image of a fuzzy soft set $f_A$.

(2) Let $g_B$ be a fuzzy soft set in $F(Y, B)$ . Then the inverse image of $g_B$ under $f_{pu}$ , written as $f_{pu}^{-1}(g_B)$, is a fuzzy soft set in $F(X, A)$ such that

$$f_{pu}^{-1}(g_B)(\alpha)(x) = \begin{cases} g(p(\alpha))(u(x)), & p(\alpha) \in B \\ 0, & \text{otherwise} \end{cases},$$

for all $x \in A$. $f_{pu}^{-1}(g_B)$ is known as a fuzzy soft inverse image of a fuzzy soft set $g_B$.

The fuzzy soft function $f_{pu}$ is called fuzzy soft surjective, if $p$ and $u$ are surjective. The fuzzy soft function $f_{pu}$ is called fuzzy soft injective, if $p$ and $u$ are injective.

**Theorem 2.12** ([1])**.** *Let $f_A, g_A \in F(X,A)$. Then for a function $f_{pu} : F(X,A) \to F(Y,B)$, the following statements are true:*

(1) $f_{pu}(\tilde{0}_A) = \tilde{0}_B$.

(2) $f_{pu}(\tilde{1}_A) = \tilde{1}_B$.

(3) $f_{pu}(f_A \tilde{\vee} g_A) = f_{pu}(f_A) \tilde{\vee} f_{pu}(g_A)$ . *In general* $f_{pu}(\tilde{\bigvee}_i((f_A)_i) = \tilde{\bigvee}_i(f_{pu}(f_A)_i)$.

(4) $f_{pu}(f_A \tilde{\wedge} g_A) \tilde{\leq} f_{pu}(f_A) \tilde{\wedge} f_{pu}(g_B)$ . *In general* $f_{pu}(\tilde{\bigwedge}_i(f_A)_i) \tilde{\leq} \tilde{\bigwedge}_i(f_{pu}(f_A)_i)$.

(5) *If* $f_A \tilde{\leq} g_A$, *then* $f_{pu}(f_A) \tilde{\leq} f_{pu}(g_A)$.

**Definition 2.13** ([36])**.** Let $(X, \tau_1, A)$ and $(Y, \tau_2, B)$ are fuzzy soft topological spaces and $f_{pu} : F(X,A) \to F(Y,B)$ be a fuzzy soft mapping. Then fuzzy soft function $f_{pu} : F(X,A) \to F(Y,B)$ is fuzzy soft pu-continuous, if for any fuzzy soft open set $g_A$ in $(Y, \tau_2, B)$, $f_{pu}^{-1}(g_A)$ is fuzzy soft open in $(X, \tau_1, A)$.

**Definition 2.14** ([36])**.** Let $(X, \tau_1, A)$ and $(Y, \tau_2, B)$ are fuzzy soft topological spaces and $f_{pu} : F(X,A) \to F(Y,B)$ be a fuzzy soft function. Then $f_{pu}$ is said to be fuzzy soft pu-open(resp. fuzzy soft pu-closed), if for any fuzzy soft open (resp. fuzzy soft closed) set $h_A$ in $(X, \tau_1, A)$, $f_{pu}(h_A)$ is fuzzy soft open(resp. fuzzy soft closed) in $(Y, \tau_2, B)$.

**Definition 2.15** ([20])**.** A fuzzy soft set $f_A$ is said to be a fuzzy soft point in $(X, \tau, A)$ denoted by $e(f_A)$, if for the element $e \tilde{\in} A$, $f(e) \tilde{\neq} \tilde{0}$ and $f(e^c) \tilde{=} \tilde{0}$, for all $e^c \in A \backslash \{e\}$.

**Definition 2.16** ([20])**.** Let $(X, \tau, A)$ be a fuzzy soft topological space over $X$ and $f_A$ be a fuzzy soft set over $X$. Then $f_A$ is called a fuzzy soft neighborhood (briefly: nbd) of the fuzzy soft point $e(f_A)$, if there exists a fuzzy soft open set $h_A$ such that $e(f_A) \tilde{\in} h_A \tilde{\leq} g_A$.

## 3. Some properties of fuzzy soft compact spaces

**Definition 3.1.** Let $(X, \tau, A)$ be a fuzzy soft topological space over $X$ and $f_A$ be nonempty fuzzy soft subset set over $X$. Then

$$\tilde{\tau}_{f_A} \tilde{=} \{v_A \tilde{\wedge} f_A, \text{ where } v_A \text{ is fuzzy soft open set in } \tilde{X}\}$$

is said to be the fuzzy soft relative topology on $f_A$ and $(f_A, \tilde{\tau}_{f_A}, A)$ is called a fuzzy soft subspace of $(X, \tau, A)$.

**Definition 3.2.** Let $(X, \tau, A)$ be a fuzzy soft topological space and $f_A$ be fuzzy soft set over $X$. A collection $\tilde{\Omega} \tilde{=}\{g_{A_i}, i \in I\}$ of fuzzy soft sets is a fuzzy soft cover of a fuzzy soft set $f_A$, if $f_A \tilde{\leq} \tilde{\bigvee}_{i\in I}(g_{A_i})$. It is fuzzy soft open cover, if each member of $\tilde{\Omega}$ is a fuzzy soft open set.

If we take $\tilde{1}_A$ instead of $f_A$, then family $\tilde{\Omega}$ of fuzzy soft sets is called a fuzzy soft cover of $(X, \tau, A)$.

A finite subcollection $\tilde{\Gamma}\tilde{=}\{g_{A_i}, i = 1, 2, ...., n\}$ of $\tilde{\Omega}$ is called a finite fuzzy soft subcover of $f_A$, if $f_A \tilde{\leq} \tilde{\vee}\{g_{A_i} : i = 1, 2, ..., n\}$.

**Definition 3.3.** Let $(X, \tau, A)$ be a fuzzy soft topological space and $f_A$ be a fuzzy soft set over $X$. Then $f_A$ is called fuzzy soft compact, if every fuzzy soft open cover of $f_A$ has a finite fuzzy soft subcover. In particular, $(X, \tau, A)$ is called fuzzy soft compact, if every fuzzy soft open cover of $(X, \tau, A)$ has a finite fuzzy soft subcover.

**Example 3.4.** Let $(X, \tau, A)$ be a fuzzy soft topological space. Then any finite fuzzy soft subset $f_A$ of $(X, \tau, A)$ is fuzzy soft compact. If we take $f_A \tilde{=} \tilde{1}_A$, then $(X, \tau, A)$ is fuzzy soft compact.

**Example 3.5.** Any infinite fuzzy soft set $f_A$ with the fuzzy soft discrete topology is not fuzzy soft compact.

Since fuzzy soft topology is fuzzy soft discrete, then $\{e(g_A)\}$ is fuzzy soft open, for each fuzzy soft point $e(g_A)$. Now $f_A \tilde{=} \tilde{\bigvee}_{e(g_A)\tilde{\in} f_A}\{e(g_A)\}$ and thus $\tilde{\Omega}\tilde{=}\{\{e(g_A)\} : e(g_A)\tilde{\in} f_A\}$ is fuzzy soft open cover of $f_A$. Since $f_A$ is infinite, this fuzzy soft open cover has no finite fuzzy soft subcover of $f_A$. Thus $f_A$ is not fuzzy soft compact. If we take $f_A \tilde{=} \tilde{1}_A$, then $(X, \tau, A)$ is fuzzy soft compact.

**Example 3.6.** Let $f_A$ be any fuzzy soft set with the cofinite fuzzy soft topology in fuzzy soft topological space $(X, \tau, A)$. Then $f_A$ is fuzzy soft compact.

Let $\tilde{\Omega} \tilde{=}\{g_{A_i}, i \in I\}$ of fuzzy soft sets is a fuzzy soft open cover of a fuzzy soft set $f_A$. Choose any $g_{A_{i_0}} \tilde{\in} \tilde{\Omega}$. Since fuzzy soft topology is cofinite, then $(g_{A_{i_0}})^c$ is finite. Let $(g_{A_{i_0}})^c \tilde{=}\{e(h_{A_1}), e(h_{A_2}), ..., e(h_{A_n})\}$. Since $f_A \tilde{=} \tilde{\bigvee}_{i\in I} (g_{A_i})$, then for each $i = 1, 2, ..., n$, there exists some $g_{A_i} \tilde{\in} \tilde{\Omega}$ such that $e(h_{A_i}) \tilde{\in} g_{A_i}$. Then

$$\begin{aligned} f_A &\tilde{=} g_{A_{i_0}} \tilde{\vee} (g_{A_{i_0}})^c \tilde{=} g_{A_{i_0}} \tilde{\vee}\{e(h_{A_1})\}\tilde{\vee}\{e(h_{A_2})\}, ..., \tilde{\vee}\{e(h_{A_n})\} \\ &\tilde{\leq} g_{A_{i_0}} \tilde{\vee} g_{A_{i_1}} \tilde{\vee} g_{A_{i_2}} \tilde{\vee} .... \tilde{\vee} g_{A_{i_n}}. \end{aligned}$$

Thus $\{g_{A_{i_0}}, g_{A_{i_1}}, g_{A_{i_2}}, ..., g_{A_{i_n}}\}$ is finite fuzzy soft subcover of $f_A$. Hence $f_A$ is fuzzy soft compact. If we take $f_A \tilde{=} \tilde{1}_A$, then $(X, \tau, A)$ is fuzzy soft compact.

**Theorem 3.7.** *Let $(X, \tau, A)$ be a fuzzy soft compact fuzzy soft spaces and $f_A$ be a fuzzy soft set over $X$. If $f_A$ is fuzzy soft closed then $f_A$ is fuzzy soft compact.*

**Proof.** Let $\{f_{A_\alpha} : \alpha \in I\}$ be fuzzy soft open cover of $f_A$. Since $f_A$ is fuzzy soft closed then $f^c_A$ is fuzzy soft open. Now $\tilde{1}_A \tilde{=} f_A \tilde{\vee} f^c_A \tilde{=} (\tilde{\bigvee}_{\alpha\in I} f_{A_\alpha}) \tilde{\vee} f^c_A$. This follows that $\{f_A^c, f_{A_\alpha}; \alpha \in I\}$ is fuzzy soft open cover of $(X, \tau, A)$. $(X, \tau, A)$ is fuzzy soft compact implies that this fuzzy soft open cover has finite fuzzy soft subcover $\{f_A^c, f_{A_{\alpha_1}}, f_{A_{\alpha_2}}, ........, f_{A_{\alpha_n}}\}$ of $(X, \tau, A)$. Now

$$\begin{aligned} f_A \tilde{=} f_A \tilde{\wedge} \tilde{1}_A \tilde{=} f_A \tilde{\wedge} [f_A^c \tilde{\vee} f_{A_{\alpha_1}} \tilde{\vee} f_{A_{\alpha_2}} \tilde{\vee} .......\tilde{\vee} f_{A_{\alpha_n}}] \\ \tilde{=} (f_A \tilde{\wedge} f_A^c) \tilde{\vee} (f_A \tilde{\wedge} f_{A_{\alpha_1}}) \tilde{\vee} (f_A \tilde{\wedge} f_{A_{\alpha_2}}) \tilde{\vee} ........ \tilde{\vee} (f_A \wedge f_{A_{\alpha_n}}) \\ \tilde{=} \tilde{0}_A \vee (f_A \tilde{\wedge} f_{A_{\alpha_1}}) \tilde{\vee} (f_A \tilde{\wedge} f_{A_{\alpha_2}}) \tilde{\vee} ........ \tilde{\vee} (f_A \wedge f_{A_{\alpha_n}}) \\ \tilde{\leq} f_{A_{\alpha_1}} \tilde{\vee} f_{A_{\alpha_2}} \tilde{\vee} ........ \tilde{\vee} f_{A_{\alpha_n}}. \end{aligned}$$

Thus $\{f_{A_{\alpha_1}}, f_{A_{\alpha_2}}, ........, f_{A_{\alpha_n}}\}$ is finite fuzzy soft subcover of $f_A$. Therefore, $f_A$ is fuzzy soft compact. Hence the proof.

The following example shows that the converse of above theorem is not true in general.

**Example 3.8.** Let $X = \{h_1, h_2, h_3\}$, $A = \{e_1, e_2\}$ and $\tau = \{\tilde{0}, \tilde{1}, (f_A)_1\}$, where $(f_A)_1$ is fuzzy soft sets over $X$, defined as follows

$f_1(e_1)(h_1) = 0.3$, $f_1(e_1)(h_2) = 0.4$, $f_1(e_1)(h_3) = 0.2$,
$f_1(e_2)(h_1) = 0.4$, $f_1(e_2)(h_2) = 0.2$, $f_1(e_2)(h_3) = 0.6$,
Then $\tau$ is a fuzzy soft topology on $X$ and hence $(X, \tau, A)$ is a fuzzy soft topological space over $X$. Let us take $f_A \tilde{=} \{\{h_{0.3}, h_{0.4}, h_{0.5}\}, \{h_{0.4}, h_{0.2}, h_{0.6}\}\}$. Then $f_A$ being finite is fuzzy soft compact. But $f_A$ is not fuzzy soft closed, since $f_A^c \tilde{=} \{\{h_{0.7}, h_{0.6}, h_{0.5}\}, \{h_{0.6}, h_{0.8}, h_{0.4}\}\}$ is not fuzzy soft open.

**Definition 3.9** ([17])**.** Let $(X, \tau, A)$ be a fuzzy soft topological space over $X$. Then $(X, \tau, A)$ is said to be fuzzy soft $T_2$-space, if for any two fuzzy soft points $e(g_A)$, $e(k_A)$ of fuzzy soft set $f_A$ in $(X, \tau, A)$ with $e(g_A) \tilde{\neq} e(k_A)$, there exist fuzzy soft open sets $h_A$ and $s_A$ such that $e(g_A) \tilde{\in} h_A$, $e(k_A) \tilde{\in} s_A$ and $h_A \tilde{\wedge} s_A \tilde{=} \tilde{0}_A$.

The converse of the above Theorem 3.7 is true, if $(X, \tau, A)$ is fuzzy soft $T_2$-space. To prove this, we need the following theorem:

**Theorem 3.10.** *Let $(X, \tau, A)$ be a fuzzy soft $T_2$-space, $f_A$ be a fuzzy soft compact subset of $(X, \tau, A)$ and fuzzy soft point $e(g_A) \tilde{\in} f_A$. Then there exist two fuzzy soft open sets $h_A$ and $s_A$ such that $e(g_A) \tilde{\in} h_A$, $f_A \tilde{\leq} s_A$ and $h_A \tilde{\wedge} s_A \tilde{=} 0_A$.*

**Proof.** Let us take fuzzy soft point $e(k_A) \tilde{\in} f_A$ with $e(g_A) \tilde{\neq} e(k_A)$. Since $(X, \tau, A)$ be a fuzzy soft $T_2$-space, then there exists fuzzy soft open sets $(h_{e(k_A)})_A$ and $(s_{e(k_A)})_A$ such that

$$(*) \qquad e(g_A) \tilde{\in} (h_{e(k_A)})_A, e(k_A) \tilde{\in} (s_{e(k_A)})_A \text{ and } (h_{e(k_A)})_A \tilde{\wedge} (s_{e(k_A)})_A \tilde{=} 0_A ...$$

Now $f_A \tilde{=} \tilde{\bigvee}_{e(k_A) \tilde{\in} f_A} \{e(k_A)\} \tilde{\leq} \tilde{\bigvee}_{e(k_A) \tilde{\in} f_A} (s_{e(k_A)})_A$, and so $\{(s_{e(k_A)})_A : e(k_A) \tilde{\in} f_A\}$ is fuzzy soft open cover of $f_A$. Since $f_A$ is fuzzy soft compact, then this fuzzy soft open cover has a finite fuzzy soft subcover $\{(s_{e(k_A)})_{A_1}, (s_{e(k_A)})_{A_2}, ..., (s_{e(k_A)})_{A_n}\}$ (say) of $f_A$. Let $(h_{e(k_A)})_{A_1}, (h_{e(k_A)})_{A_2}, ..., (h_{e(k_A)})_{A_n}$ be the fuzzy soft sets corresponding to $(s_{e(k_A)})_{A_1}, (s_{e(k_A)})_{A_2}, ..., (s_{e(k_A)})_{A_n}$ which satisfy (*).

Put $h_A \tilde{=} \tilde{\wedge}\{(h_{e(k_A)})_{A_i} : i = 1, 2, ..., n\}$, $s_A \tilde{=} \tilde{\vee}\{(s_{e(k_A)})_{A_i} : i = 1, 2, ..., n\}$. Clearly, $h_A$ and $s_A$ are fuzzy soft open. Further, $e(g_A)\tilde{\in} h_A$, since $e(g_A)\tilde{\in}(h_{e(k_A)})_{A_i}$, for each $i = 1, 2, ..., n$.

Moreover, $f_A \tilde{\leq} s_A$, since $\{(s_{e(k_A)})_{A_1}, (s_{e(k_A)})_{A_2}, ..., (s_{e(k_A)})_{A_n}\}$ is finite fuzzy soft subcover of $f_A$. We claim that $h_A \tilde{\wedge} s_A \tilde{=} 0_A$. To verify the claim, we contrarily suppose that $h_A \tilde{\wedge} s_A \tilde{\neq} 0_A$ and let $e(l_A)\tilde{\in} h_A \tilde{\wedge} s_A$.

Then, we have $e(l_A)\tilde{\in}\tilde{\wedge}\{(h_{e(k_A)})_{A_i} : i = 1, 2, ..., n\}$ and $e(l_A)\tilde{\in}\tilde{\vee}\{(s_{e(k_A)})_{A_i} : i = 1, 2, ..., n\}$. This implies that $e(l_A)\tilde{\in}(h_{e(k_A)})_{A_i}$, for each $i = 1, 2, ..., n$ and $e(l_A)\tilde{\in}(s_{e(k_A)})_{A_i}$, for some $i = 1, 2, ..., n$.

This follows that $(h_{e(k_A)})_{A_i} \tilde{\wedge} (s_{e(k_A)})_{A_i} \tilde{\neq} 0_A$. This contradicts (*). Hence $h_A \tilde{\wedge} s_A \tilde{=} 0_A$. This completes the proof.

Using above Theorem 3.10, we have the following.

**Theorem 3.11.** *Let $(X, \tau, A)$ be a fuzzy soft $T_2$-space, $f_A$ be a fuzzy soft subset of $(X, \tau, A)$. If $f_A$ is fuzzy soft compact then $f_A$ is fuzzy soft closed.*

**Proof.** To show that $f_A$ is fuzzy soft closed, we prove that $(f_A)^c$ is fuzzy soft open. Let $e(g_A)\tilde{\in}(f_A)^c$. Then $e(g_A)\tilde{\notin} f_A$ and $f_A$ is fuzzy soft compact. So, by Theorem 3.10, there exist two fuzzy soft open sets $h_A$ and $s_A$ such that $e(g_A)\tilde{\in} h_A$, $f_A \tilde{\leq} s_A$ and $h_A \tilde{\wedge} s_A \tilde{=} 0_A$. This follows that $h_A \tilde{\leq} (s_A)^c \tilde{\leq} (f_A)^c$. Therefore, $e(g_A)\tilde{\in} h_A \tilde{\leq} (f_A)^c$. This implies that $(f_A)^c$ is fuzzy soft open and therefore $f_A$ is fuzzy soft closed. Hence the proof.

**Definition 3.12.** Let $(X, \tau, A)$ be a fuzzy soft topological space and $f_A$ be a fuzzy soft set over $X$. A collection $\tilde{\Omega} \tilde{=} \{g_{A_i}, i \in I\}$ of fuzzy soft subsets of $X$ is said to have the finite intersection property, if for any finite fuzzy soft subcollection $\{g_{A_i}, i = 1, 2, ...., n\}$ of $\tilde{\Omega}$, we have $\tilde{\wedge}\{g_{A_i} : i = 1, 2, ..., n\} \tilde{\neq} \tilde{0}_A$.

The following theorem gives the useful characterization of fuzzy soft compact spaces.

**Theorem 3.13.** *Let $(X, \tau, A)$ be a fuzzy soft topological space. Then the following statements are equivalent:*

(1) *$(X, \tau, A)$ is fuzzy soft compact.*

(2) *For any collection $\{g_{A_i}, i \in I\}$ of fuzzy soft closed subsets of $(X, \tau, A)$ which satisfy the finite intersection property implies $\tilde{\bigwedge}_{i\in I}(g_{A_i}) \tilde{\neq} 0_A$.*

**Proof.** (1) $\Rightarrow$ (2) Let $(X, \tau, A)$ is fuzzy soft compact and $\{g_{A_i}, i \in I\}$ be any collection of fuzzy soft closed subsets of $(X, \tau, A)$ with the finite intersection property. We show that $\tilde{\bigwedge}_{i\in I}(g_{A_i}) \tilde{\neq} 0_A$. For this contrarily suppose that $\tilde{\bigwedge}_{i\in I}(g_{A_i}) \tilde{=} 0_A$. Then $(\tilde{\bigwedge}_{i\in I}(g_{A_i}))^c \tilde{=} (\tilde{0}_A)^c$ or $\tilde{\bigvee}_{i\in I}(g_{A_i})^c \tilde{=} \tilde{1}_A$. Thus $\{(g_{A_i})^c : i \in I\}$ is a fuzzy soft open cover for $(X, \tau, A)$.

Since $(X, \tau, A)$ is fuzzy soft compact, then this fuzzy soft open cover has a finite fuzzy soft subcover, $\{(g_{A_{i_1}})^c, (g_{A_{i_2}})^c, ..., (g_{A_{i_n}})^c\}$. Then $\tilde{\bigvee}_{k=1}^{n}(g_{A_{i_k}})^c \tilde{=} \tilde{1}_A$

or $(\tilde{\bigwedge}_{k=1}^{n}(g_{A_{i_k}}))^c \tilde{=} \tilde{1}_A$ or $\tilde{\bigwedge}_{k=1}^{n}(g_{A_{i_k}}) \tilde{=} \tilde{0}_A$. This contradicts the fact that $\{g_{A_i}, i \in I\}$ satisfy the finite intersection property. Thus $\tilde{\bigwedge}_{i\in I}(g_{A_i}) \tilde{\neq} 0_A$.

(2) $\Rightarrow$ (1) Suppose (2) holds and let $\{h_{A_i}, i \in I\}$ be any fuzzy soft open cover of $(X, \tau, A)$. Contrarily suppose that $\{h_{A_i}, i \in I\}$ has no finite fuzzy soft subcover of $(X, \tau, A)$. Then for every finite subcollection $\{h_{A_{i_1}}, h_{A_{i_2}}, ..., h_{A_{i_n}}\}$ of $\{h_{A_i}, i \in I\}$, $\tilde{\bigvee}_{k=1}^{n}(h_{A_{i_k}}) \tilde{\neq} \tilde{1}_A$ or $(\tilde{\bigvee}_{k=1}^{n}(h_{A_{i_k}}))^c \tilde{\neq} \tilde{0}_A$ or $\tilde{\bigwedge}_{k=1}^{n}(h_{A_{i_k}})^c \tilde{\neq} \tilde{0}_A$. Thus $\{(h_{A_i})^c, i \in I\}$ is a collection of fuzzy soft closed sets with the finite intersection property. Hence by (2), $\tilde{\bigwedge}_{i\in I}(h_{A_i})^c \tilde{\neq} \tilde{0}_A$ or $(\tilde{\bigvee}_{i\in I}(h_{A_i}))^c \tilde{\neq} \tilde{0}_A$ or $\tilde{\bigvee}_{i\in I}(h_{A_i}) \tilde{\neq} \tilde{1}_A$. This contradicts the fact that $\{h_{A_i}, i \in I\}$ is fuzzy soft open cover of $(X, \tau, A)$. Thus $\{h_{A_i}, i \in I\}$ has finite fuzzy soft subcover of $(X, \tau, A)$. Therefore, $(X, \tau, A)$ is fuzzy soft compact. This completes the proof.

**Theorem 3.14.** *Let $(X, \tau_1, A)$ be a fuzzy soft compact fuzzy soft topological space and $(Y, \tau_2, B)$ be a fuzzy soft topological spaces. Suppose fuzzy soft function $f_{pu} : F(X, A) \to F(Y, B)$ be fuzzy soft pu-continuous, fuzzy soft surjective function form fuzzy soft classes $F(X, A)$ of $(X, \tau, A)$ to the fuzzy soft classes $F(Y, B)$ of $(Y, \tau_2, B)$. Then $(Y, \tau_2, B)$ is fuzzy soft compact.*

**Proof.** Let $\tilde{\Psi} \tilde{=} \{g_{A_i}, i \in I\}$ be a fuzzy soft open cover of $(Y, \tau_2, B)$. Since $f_{pu}$ is fuzzy soft pu-continuous, the family of all fuzzy soft sets of the form $\{h_{A_i} : h_{A_i} \tilde{=} f_{pu}^{-1}(g_{A_i}), i \in I\}$, is a fuzzy soft open cover of $(X, \tau_1, A)$. Since $(X, \tau_1, A)$ is fuzzy soft compact, then there are indices $i_1, i_2, ..., i_n$ such that $\tilde{1}_A \tilde{=} \tilde{\bigvee}_{k=1}^{n} h_{A_{i_k}}$. Now $f_{pu}(\tilde{1}_A) \tilde{=} f_{pu}(\tilde{\bigvee}_{k=1}^{n} h_{A_{i_k}}) \tilde{=} \tilde{\bigvee}_{k=1}^{n} f_{pu}(h_{A_{i_k}}) \tilde{=} \tilde{\bigvee}_{k=1}^{n}(g_{A_{i_k}}) \tilde{\leq} f_{pu}(\tilde{1}_A)$. This follows that $f_{pu}(\tilde{1}_A) \tilde{=} \tilde{\bigvee}_{k=1}^{n}(g_{A_i})$. Thus $(Y, \tau_2, B)$ is fuzzy soft compact. Hence the proof.

**Definition 3.15** ([28]). A fuzzy soft topological space $(X, \tau, A)$ is said to be fuzzy soft normal, if for any two fuzzy soft closed set $f_A$ and $g_A$ in $(X, \tau, A)$ with $f_A \tilde{\wedge} g_A \tilde{=} \tilde{0}_A$, there are fuzzy soft open sets $h_A$ and $k_A$ such that $f_A \tilde{\leq} h_A$, $g_A \tilde{\leq} k_A$ and $f_A \tilde{\wedge} g_A \tilde{=} \tilde{0}_A$.

**Theorem 3.16.** *Let $(X, \tau, A)$ be a fuzzy soft topological space. If $(X, \tau, A)$ is a fuzzy soft compact and fuzzy soft $T_2$ space then it is fuzzy soft normal.*

**Proof.** Let $(X, \tau, A)$ be a fuzzy soft compact and fuzzy soft $T_2$-space. Let $f_A$ and $k_A$ are any two fuzzy soft closed and fuzzy soft disjoint subsets of $(X, \tau, A)$. Then by Theorem 3.10, for any $e(g_A) \tilde{\in} f_A$ there exist two fuzzy soft open sets $h_A$ and $s_A$ such that $e(g_A) \tilde{\in} h_A$, $k_A \tilde{\leq} s_A$ and $h_A \tilde{\wedge} s_A \tilde{=} 0_A$. The collection $\{(h_{e(g_A)})_A : e(g_A) \tilde{\in} f_A\}$ forms fuzzy soft open cover for $f_A$. Since $f_A$ is fuzzy soft closed, then by Theorem 3.7, $f_A$ is fuzzy soft compact. So there exists finite fuzzy soft subcover of $f_A$. That is, $f_A \tilde{\leq} \tilde{\bigvee}_{i=1}^{n}(h_{e(g_A)})_{A_i}$. Let $l_A \tilde{=} \tilde{\bigvee}_{i=1}^{n}(h_{e(g_A)})_{A_i}$, $m_A \tilde{=} \tilde{\bigwedge}_{i=1}^{n}(s_{e(g_A)})_{A_i}$. Then $f_A \tilde{\leq} l_A$, $k_A \tilde{\leq} m_A$ and $l_A \tilde{\wedge} m_A \tilde{=} \tilde{0}_A$. Hence $(X, \tau, A)$ is fuzzy soft normal. This completes the proof.

## 4. Fuzzy soft locally compact spaces

**Definition 4.1.** Let $(X, \tau, A)$ be a fuzzy soft topological space and $f_A$ be any fuzzy soft set in $(X, \tau, A)$. Then $f_A$ is said to be fuzzy soft locally compact, if for any fuzzy soft point $e(g_A)\tilde{\in} f_A$, there exist a fuzzy soft compact set $h_A$ and a fuzzy soft open set $k_A$ such that $e(g_A)\tilde{\in} k_A \tilde{\leq} h_A$. That is, each fuzzy soft point $e(g_A)\tilde{\in} f_A$ has a fuzzy soft compact nbd.

If we take $\tilde{1}_A$ instead of $f_A$, then $(X, \tau, A)$ is called fuzzy soft locally compact.

**Example 4.2.** Let $X$ be any set, $A$ be the set of any parameters and $\tau_D$ is a fuzzy soft discrete topology on $X$, then $(X, \tau_D, A)$ is a fuzzy soft discrete topological space over $X$. Clearly $(X, \tau_D, A)$ is fuzzy soft locally compact space. Because, if we take any fuzzy soft point $e(f_A)$ in $(X, \tau_D, A)$, $k_A \tilde{=} \{e(f_A)\}$ and $g_A \tilde{=} \{e(f_A)\}$. Then $k_A$ is fuzzy soft compact, $g_A$ is fuzzy soft open and $e(f_A)\tilde{\in} g_A \tilde{\leq} k_A$. Hence $(X, \tau_D, A)$ is fuzzy soft locally compact.

**Example 4.3.** Let $X$ be any set, $A$ be the set of any parameters and $\tau_I$ is a fuzzy soft indiscrete topology on $X$, then $(X, \tau_I, A)$ is a fuzzy soft indiscrete topological space over $X$. Clearly $(X, \tau_I, A)$ is fuzzy soft locally connected space. Because, if we take any fuzzy soft point $e(f_A)$ in $(X, \tau_I, A)$, $k_A \tilde{=} \tilde{1}_A$ and $g_A \tilde{=} \tilde{1}_A$. Then $k_A$ is fuzzy soft compact, $g_A$ is fuzzy soft open and $e(f_A)\tilde{\in} g_A \tilde{\leq} k_A$. Hence $(X, \tau_I, A)$ is fuzzy soft locally compact.

**Theorem 4.4.** *Let $(X, \tau, A)$ be a fuzzy soft locally compact fuzzy soft space. If $f_A$ be a fuzzy soft closed subset of $(X, \tau, A)$. Then $f_A$ is fuzzy soft locally compact.*

**Proof.** Let $f_A$ be a fuzzy soft closed set in a fuzzy soft locally compact space $(X, \tau, A)$. We show that $f_A$ is fuzzy soft locally compact. Let us take fuzzy soft point $e(g_A)\tilde{\in} f_A$. Then $e(g_A)$ is also fuzzy soft point in $(X, \tau, A)$. Since $(X, \tau, A)$ is fuzzy soft locally compact, then there exists a fuzzy soft compact set $k_A$ and fuzzy soft open set $s_A$ such that $e(g_A)\tilde{\in} s_A \tilde{\leq} k_A$. Take $h_A \tilde{=} f_A \tilde{\wedge} k_A$ and $v_A \tilde{=} f_A \tilde{\wedge} s_A$. Then $e(g_A)\tilde{\in} v_A \tilde{\leq} h_A$. Since $f_A$ is fuzzy soft closed, $f_A \tilde{\wedge} k_A$ is fuzzy soft closed subset of fuzzy soft compact set $k_A$ and thus $f_A \tilde{\wedge} k_A$ is fuzzy soft compact in $k_A$. Hence $h_A \tilde{=} f_A \tilde{\wedge} k_A$ is also fuzzy soft compact in $f_A$. Clearly, $v_A$ is fuzzy soft open in $f_A$, since $s_A$ is fuzzy soft open in $(X, \tau, A)$. Thus $f_A$ is fuzzy soft locally compact. This completes the proof.

**Theorem 4.5.** *Let $(X, \tau_1, A)$ and $(Y, \tau_2, B)$ are two fuzzy soft topological spaces and $u : X \to Y$ and $p : A \to B$ be mappings such that $(X, \tau_1, A)$ be a fuzzy soft locally compact space. If $f_{pu} : F(X, A) \to F(Y, B)$ be fuzzy soft pu-continuous, fuzzy soft pu-open and fuzzy soft surjective then $(Y, \tau_2, B)$ is fuzzy soft locally compact.*

**Proof.** To show that $(Y, \tau_2, B)$ is fuzzy soft locally compact, let us take fuzzy soft point $e(g_B)$ in $(Y, \tau_2, B)$. Since $f_{pu}$ is fuzzy soft surjective, then there exist a fuzzy soft point $e(h_A)$ in $(X, \tau_1, A)$ such that $e(g_B)\tilde{=}f_{pu}(e(h_A))$. Also $(X, \tau_1, A)$ is fuzzy soft locally compact implies that there exists a fuzzy soft compact set $k_A$ in $(X, \tau_1, A)$ and fuzzy soft open set $v_A$ in $(X, \tau_1, A)$ such that $e(h_A)\tilde{\in}v_A\tilde{\leq}k_A$. This implies that $e(g_B)\tilde{=}f_{pu}(e(h_A))\tilde{\in}f_{pu}(v_A)\tilde{\leq}f_{pu}(k_A)$. Further, $f_{pu}(v_A)$ is fuzzy soft open, since $v_A$ is fuzzy soft open in $(X, \tau_1, A)$ and $f_{pu}$ is fuzzy soft pu-open. Moreover, $f_{pu}(k_A)$ is fuzzy soft compact in $(Y, \tau_2, B)$, since $k_A$ is fuzzy soft compact in $(X, \tau_1, A)$ and $f_{pu}$ is fuzzy soft pu-continuous. Thus $(Y, \tau_2, B)$ is fuzzy soft locally compact. Hence the proof.

**Theorem 4.6.** *Let $(X, \tau, A)$ be a fuzzy soft locally compact and fuzzy soft $T_2$ fuzzy soft space. Then for any fuzzy soft point $e(f_A)$ in $(X, \tau, A)$, there exists a fuzzy soft open set $g_A$ such that $e(f_A)\tilde{\in}g_A$ and $\overline{g_A}$ is fuzzy soft compact.*

**Proof.** Let $e(f_A)$ be a fuzzy soft point in $(X, \tau, A)$. Since $(X, \tau, A)$ is fuzzy soft locally compact, then there exists a fuzzy soft compact set $k_A$ and a fuzzy soft open set $g_A$ such that $e(f_A)\tilde{\in}g_A\tilde{\leq}k_A$. Now we prove that $\overline{g_A}$ is fuzzy soft compact. Since $k_A$ is a fuzzy soft compact subset of fuzzy soft $T_2$ space $(X, \tau, A)$, then by Theorem 3.11, $k_A$ is fuzzy soft closed. Thus $\overline{g_A}\tilde{\leq}\overline{k_A}\tilde{=}k_A$. Hence by Theorem 3.7, $\overline{g_A}$ being a fuzzy closed subset of a fuzzy soft compact set $k_A$ is fuzzy soft compact. This completes the proof.

**Theorem 4.7.** *Let $(X, \tau, A)$ be a fuzzy soft locally compact, fuzzy soft $T_2$ space and $h_A$ be a fuzzy soft subset in $(X, \tau, A)$. If $h_A$ is fuzzy soft open, then $h_A$ is fuzzy soft locally compact.*

**Proof.** Let $e(f_A)$ be a fuzzy soft point in $h_A$. By above Theorem 4.6, there exists a fuzzy soft open set $g_A$ such that $e(f_A)\tilde{\in}g_A$ and $\overline{g_A}$ is fuzzy soft compact. Take $v_A\tilde{=}g_A\tilde{\wedge}h_A$ and $k_A\tilde{=}\overline{g_A}\tilde{\wedge}h_A$. Then $e(f_A)\tilde{\in}v_A\tilde{\leq}k_A$. Clearly, $v_A$ is fuzzy soft open in $h_A$ and $k_A$ is fuzzy soft compact in $h_A$. Therefore, $h_A$ is fuzzy soft locally compact. Hence the proof.

Now we give the characterization of fuzzy soft locally compact spaces.

**Theorem 4.8.** *Let $(X, \tau, A)$ be a fuzzy soft $T_2$ fuzzy soft space. Then the following statements are equivalent:*

(1) $(X, \tau, A)$ *is fuzzy soft locally compact.*

(2) *For any fuzzy soft compact set $k_A$ in $(X, \tau, A)$, there exists a fuzzy soft open set $v_A$ such that $k_A\tilde{\leq}v_A$ and $\overline{v_A}$ is fuzzy soft compact.*

**Proof.** (1) $\Rightarrow$ (2) Given $(X, \tau, A)$ is fuzzy soft locally compact and fuzzy soft $T_2$ fuzzy soft space. Then by Theorem 4.6, for any fuzzy soft point $e(f_A)$ in $(X, \tau, A)$, there exists a fuzzy soft open set $(v_{e(f_A)})_A$ such that $e(f_A)\tilde{\in}(v_{e(f_A)})_A$ and $\overline{(v_{e(f_A)})_A}$ is fuzzy soft compact. Since $k_A$ is fuzzy soft compact, then the fuzzy soft open cover $\{(v_{e(f_A)})_A : e(f_A)\tilde{\in}k_A\}$ has a finite fuzzy soft subcover $\{(v_{e(f_A)})_{A_1}, (v_{e(f_A)})_{A_2}, ..., (v_{e(f_A)})_{A_n}\}$ (say). Let $v_A\tilde{=}\tilde{\bigvee}_{i=1}^{n}(v_{e(f_A)})_{A_i}$. Clearly, $v_A$ is fuzzy soft open, $k_A\tilde{\leq}v_A$ and $\overline{v_A}$ is fuzzy soft compact.

(2) $\Rightarrow$ (1) For any fuzzy soft point $e(f_A)$ in $(X, \tau, A)$, take $k_A \tilde{=} \{e(f_A)\}$. Then there exists a fuzzy soft open set $v_A$ such that $\{e(f_A)\} \tilde{\leq} v_A$ with $\overline{v_A}$ is fuzzy soft compact. Hence $(X, \tau, A)$ is fuzzy soft locally compact. This completes the proof.

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Accepted: 29.03.2019